

Dynamics Control and Instrumentation Assignment



Matthew Williams

Gower College Swansea

Tycoch Campus

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Course Number & Name

Linton Cook

Abstract

Solve, analyze and simulate the behavior of dynamic mechanical systems using traditional analytical techniques and computer based simulation.

Apply instrumentation and control system methodologies to dynamical systems and analyze the behavior and response of the system and how it can be controlled.

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1 Task 1

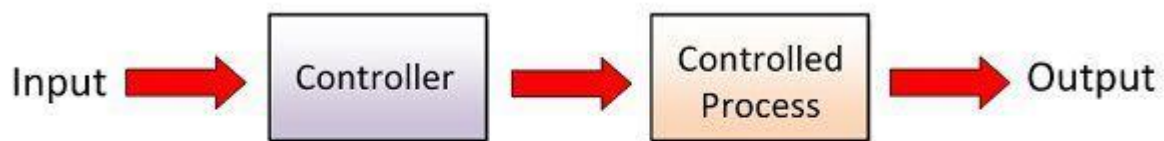
1.1 The key differences between Open Loop and a Closed Loop control systems.

1. The open loop system means the output of the system is free from their input. In the closed-loop system, the desired output depends on their input.
2. The open loop system is called the non-feedback system while the closed loop is the feedback system.
3. The control and controlled process are the two components of the open loop system. The closed loop requires some components likes an amplifier, controller, controlled process, feedback system etc.
4. The construction of systems is easy because few elements are used in the system. The construction of the closed-loop system is quite difficult.
5. The open loop system is not reliable whereas the closed-loop system is reliable.
6. The accuracy of the system is less as compared to the closed-loop system.
7. The open loop system is more stable as compared to a closed loop system. Here the word stable means the output of the system remains constant even after the disturbances.
8. The open loop system is not optimized, whereas the closed-loop system is optimized.
9. The open loop system gives the fast response, whereas the closed loop system gives the slow response.
10. The calibration of open loop system is difficult as compared to the closed-loop system.
11. In an open loop system, the disturbance affected the output, whereas in a closed loop system the output is not much affected by the disturbances.
12. The output control system has a non-linear response, whereas the input control system has linear responses.
13. The traffic light, automatic washing machine, etc. are the examples of the output system, whereas the temperature controller, toaster etc. are the examples of the closed-loop system. (Author, 2021)

1.2 The operational differences between Open and Closed Loop systems.

1.2.1 Open Loop Control Systems

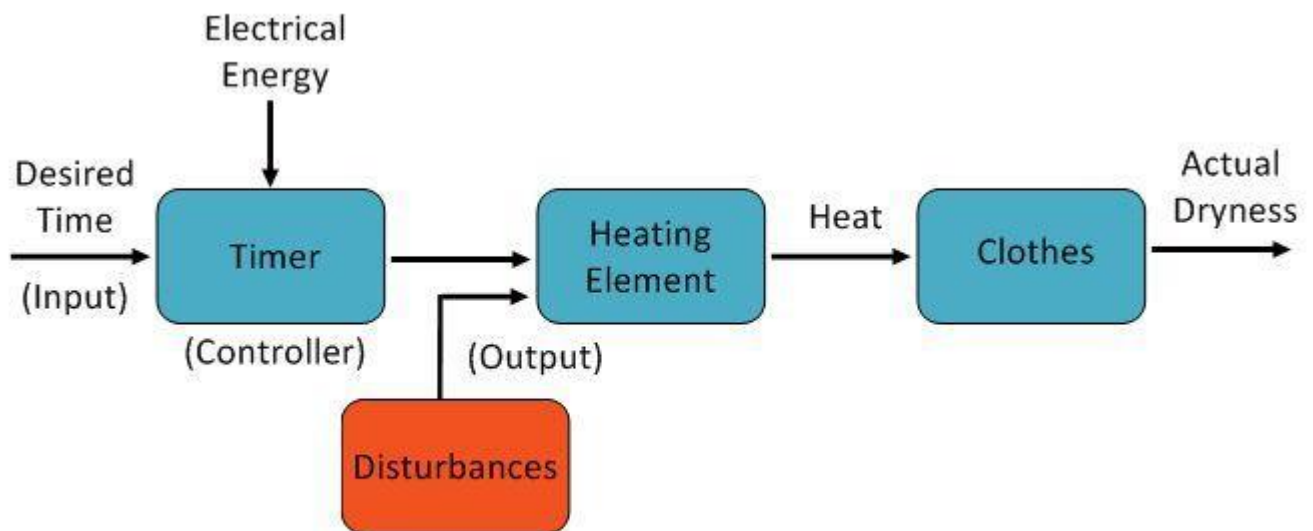
In open loop control system, the output does not affect the control action of the system. In other words, the system whose working depends on time is known as the open loop control system. The open loop system is free from the feedback. Let's understand this with the help of the few examples.



Open Loop System

Circuit Globe

Example 1: Consider the clothes dryer whose control action is done manually by the operator. Depending on the wetness of the clothes suppose the operator set the timer for 30 minutes. After 30 minutes the timer will stop even after the clothes are wet.



Circuit Globe

The dryer stops working even if the desired output is not obtained. This shows that the system has no feedback. Here clothes dryer is the example of the open loop system and the timer is the controller of the system.

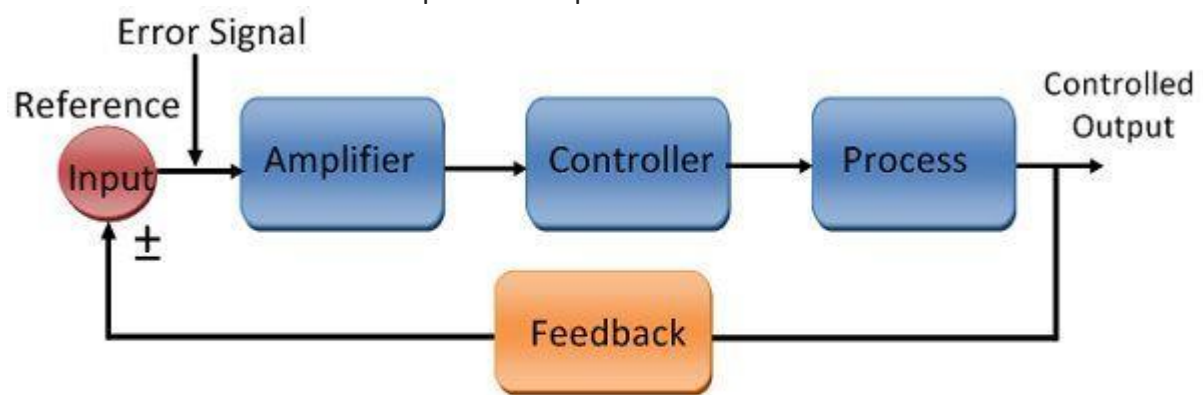
Example 2: The automatic washing machine is the example of the open loop system. The operator manually sets the operating time of the machine. The machine stops operating after the set time, even the desired cleanliness of clothes are not obtained. This happens because the machine has no feedback system which signals the control action of the system for desired output.

The open loop system is simple, requires less maintenance. Also, it is fast in operation and very economical. But the accuracy of the system is less, and it is less reliable.

$$\frac{\text{Output}}{\text{Input}} = \text{Gain}$$

1.2.2 Closed Loop Control System

The closed-loop control system means the output of the system depends on their input. The system has one or more feedback loops between its output and input. The closed-loop system is designed in such a way that they automatically provide the desired output by comparing it with the actual input. The closed-loop system generates the error signal which is the difference between the input and output.

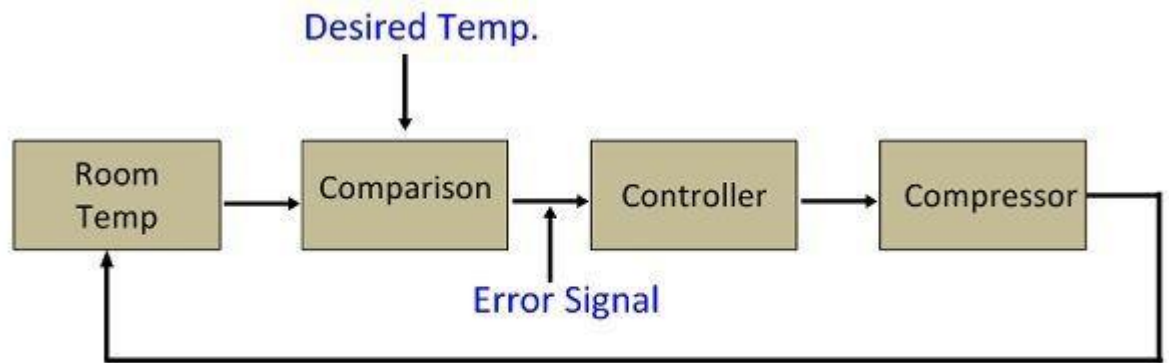


Closed Loop Control System

Circuit Globe

Example 1: Suppose in the above example of closed dryer we are using the transducer which senses the dryness of the clothes and provides the feedback signal to the controller relating to dryness. Here the dryness is the output of the system. The sensor acts as a feedback of the system. The sensor gives the signal to the controller of the machine, and hence the dryer provides the desired output.

Example 2: The air conditioner is the example of the closed-loop system. The air conditioner regulates the temperature by comparing it with the surrounding temperature. The comparison of temperature is done with the help of the thermostat. When the AC provides the error signal which is the difference between the surrounding temperature and room temperature the thermostats turn on or off the compressor.



Circuit Globe

The closed loop system is more reliable and accurate. But this system is very expensive and requires high maintenance.

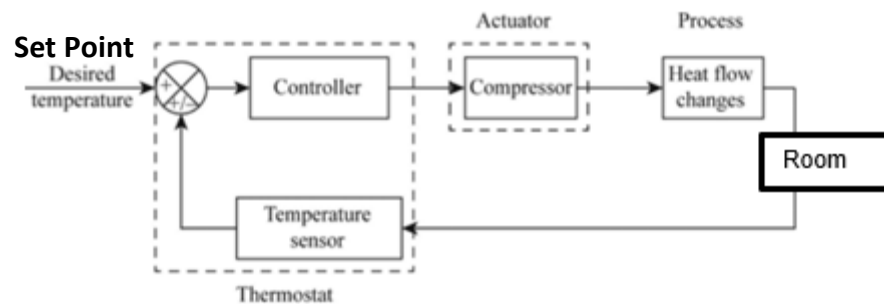
$$X(s) \rightarrow \frac{G(s)}{1 + G(s)H(s)} \rightarrow Y(s)$$

(Author, 2021)

1.3 Explanations of common terms within Control Systems.

1.3.1 Set Point

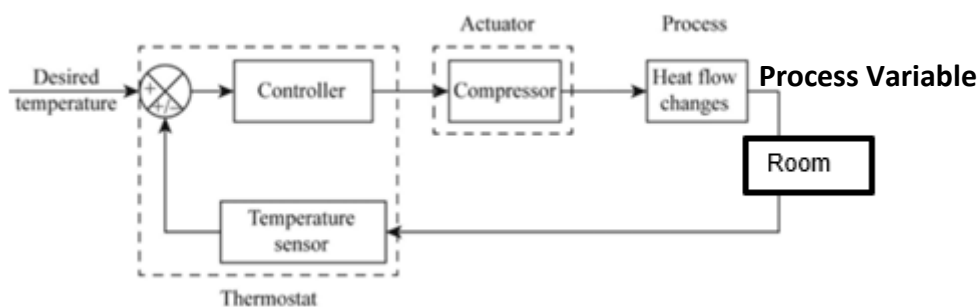
A set point is the target value in a variable process system. E.g. in a temperature control system, the set point is the target temperature that the system controller (room stat) attempts to maintain the process.



1.3.2 Process variable

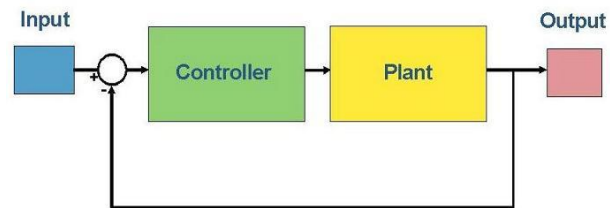
The Process variable is the measured value of a part of the system that changes over time.

E.g. in the temperature control system, the process variable is the Heat change in the room, measured by the Temperature sensor.



1.3.3 Plant

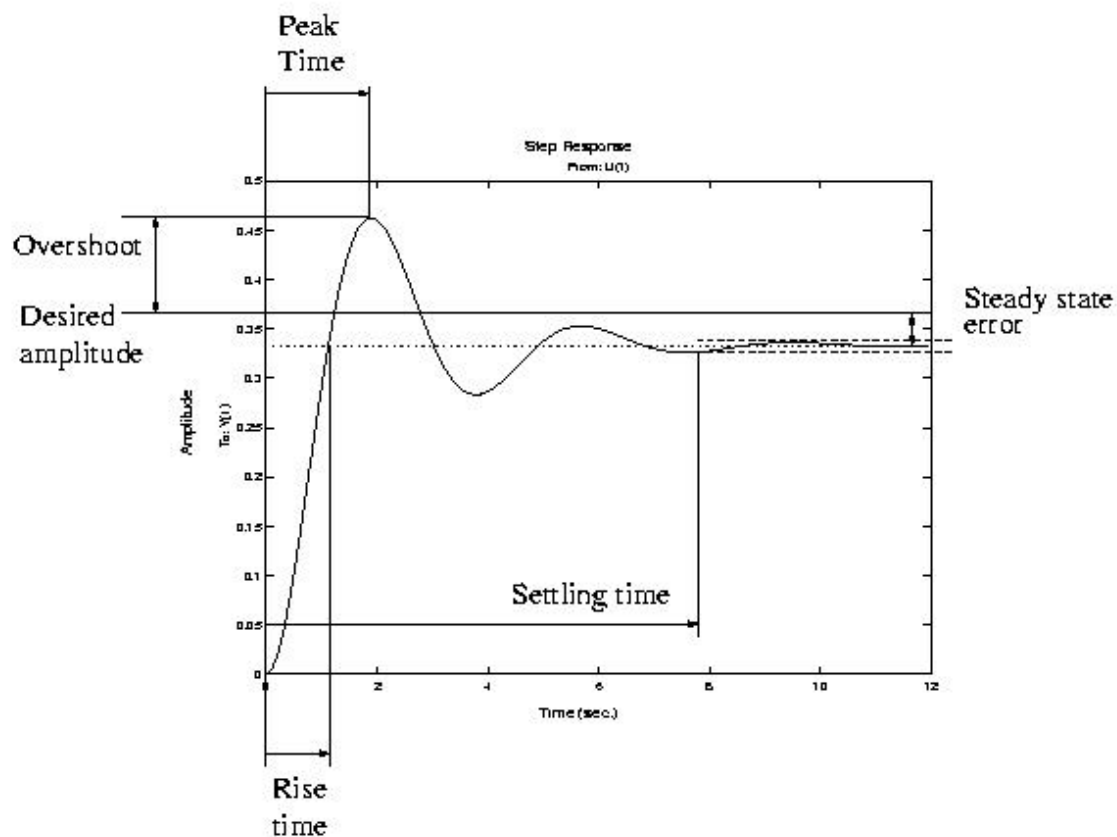
The Plant part of the control system is simply “The thing that’s being controlled”. E.g. in our temperature control system, the plant part is the compressor generating heat, the heat change in the room and the Temperature sensor that is measuring the variable.



(Radhesh, 2021)

1.3.4 Settling Time

The Settling Time is the time it takes a system to reach a steady state from the beginning of the response time.



(B. Goodwine, 2000)

1.3.5 Steady state error

The Steady State Error is the difference between the desired output of a system, and the actual output of a system once the system has reached the steady state. (See above)

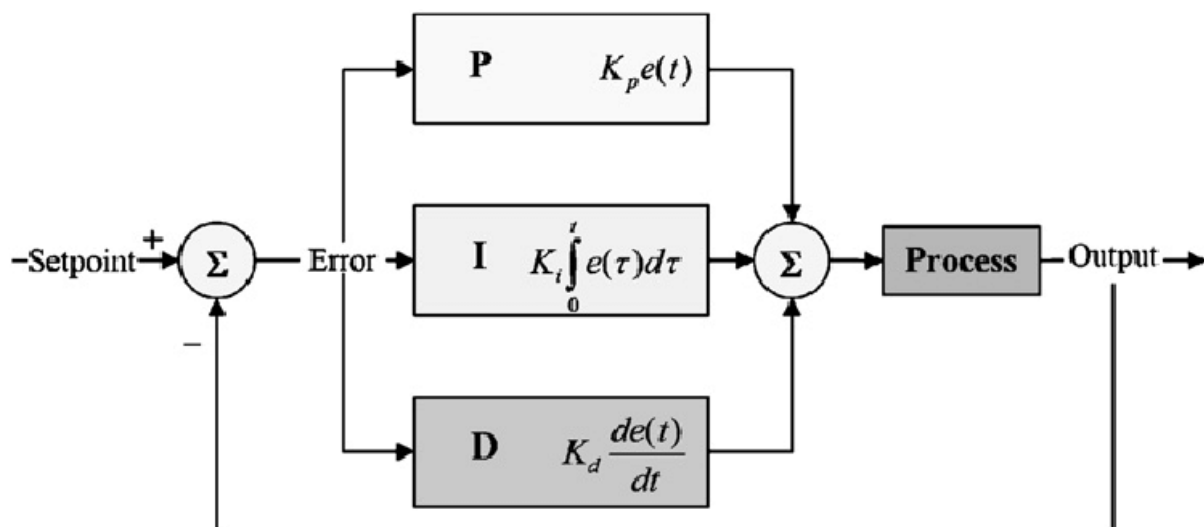
1.3.6 PID

A PID (Proportional-Integral-Differential) controller, is an instrument used in industrial control applications to regulate temperature, flow, pressure, speed and other process variables. PID controllers use a control loop feedback mechanism to control process variables and are the most accurate and stable controllers. PID controllers are made up of three parts that controls a system so that the output signal is as close to the Set Point in the best way possible.

P –Proportional - Amplifier or Gain

I – Integral - Helps to eliminate Steady State Error

D – Differential - Slows the rate of change as it gets to the Set Point



(Das, 21)

2 Task 2

2.1 Dynamic systems stability, and methods of mathematically analysing the stability.

2.1.1 Stable System

A Stable System stabilises to zero as time approaches Infinity. This is stable as any signal into the Transfer Function will ultimately die out.

$f(t)$ (function over time) $\rightarrow$ goes to 0 (zero) $\rightarrow$ as time goes to ∞ (infinity)

2.1.2 Unstable System

An Unstable System does not settle to a zero point but goes off into a positive or negative infinity over time

$f(t)$ $\rightarrow$ goes to ∞ $\rightarrow$ as time ∞

2.1.3 Marginally Stable System

A Marginally Stable System does not reach a constant zero point over time, but neither does it go off into a positive or negative infinity. Instead, it Oscillates at a continuous frequency or stabilizes to a single point that is not zero over time

$f(t)$ does not go to 0, nor go to positive or negative ∞ $\rightarrow$ as time ∞

2.2 Poles and Zero's

Using the Poles and Zeroes of a Transfer Function is one method of analyzing and determining the stability of a system.

Within a Transfer Function you have a Numerator & Denominator:

Zero's: Value of S that makes the Transfer Function $\rightarrow$ (go to) 0 (zero)

The value of S of the Numerator that make the expression Zero

Denoted as: - **0**

Pole: Value of S that makes the Transfer Function $\rightarrow$ (go to) ∞ (infinity)

The value of S of the Denominator that make the expression Zero

Denoted as: - **X**

Example 1:

$$G(s) = \frac{S + 2}{S^2 + 5S}$$

$$G(s) = \frac{S + 2}{S^2 + 5S} \text{ First find the Zero's from the Numerator}$$

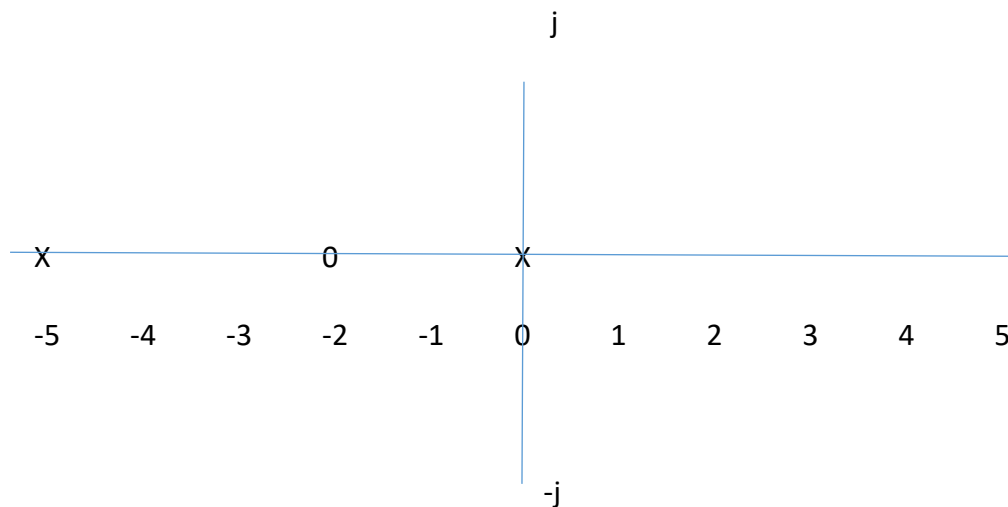
$$G(s) = \frac{S + 2}{S^2 + 5S} \quad S + 2 = 0 \quad \mathbf{S = -2 (Zero)}$$

$$G(s) = \frac{S + 2}{S^2 + 5S} \text{ Now find the Poles from the Denominator}$$

$$G(s) = \frac{S + 2}{S^2 + 5S} \quad S^2 + 5S = 0 \quad S(S + 5) = 0 \quad \mathbf{S = 0} \quad \& \quad \mathbf{S = -5 (Pole)}$$

Zero = -2 Poles = 0, -5

Pole Zero Diagram



When any of the POLES (X's) are on the center axis, we can deter that the system is Marginally Stable. (Kim, 2014)

Example 2:

$$G(s) = \frac{(S + 2)(S - 1)}{(S + 3)(S - 7)}$$

$$G(s) = \frac{(S + 2)(S - 1)}{(S + 3)(S - 7)} \text{ First find the Zero's}$$

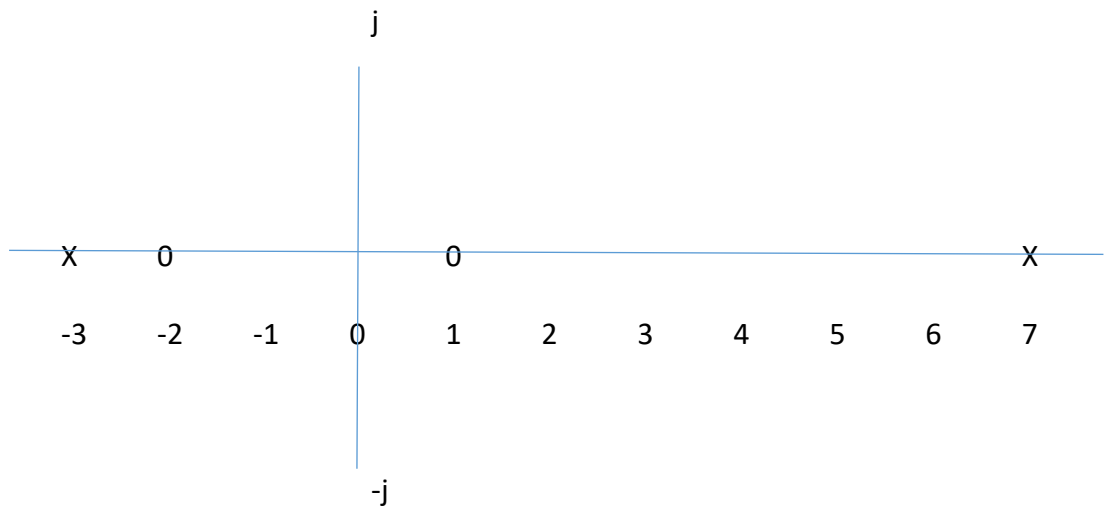
$$G(s) = \frac{(S + 2)(S - 1)}{(S + 3)(S - 7)} \quad (S + 2)(S - 1) = 0 \quad \mathbf{S = -2} \quad \& \quad \mathbf{S = +1}$$

$$G(s) = \frac{(S + 2)(S - 1)}{(S + 3)(S - 7)} \text{ Now find the Poles}$$

$$G(s) = \frac{(S + 2)(S - 1)}{(S + 3)(S - 7)} \quad (S + 3)(S - 7) = 0 \quad \mathbf{S = -3} \quad \& \quad \mathbf{S = +7}$$

Zero = -2, +1 Poles = -3, +7

Pole Zero Diagram



When any of the POLES (X's) are in the right hand plane, (right hand side of the Pole Zero Diagram) we can deter that the system is Unstable. (Cook, 2020-2021)

Example 3:

$$G(s) = \frac{(S + 2)(S + 3)}{(S + 1)(S + 4)}$$

$$G(s) = \frac{(S + 2)(S + 3)}{(S + 1)(S + 4)} \text{ First find the Zero's}$$

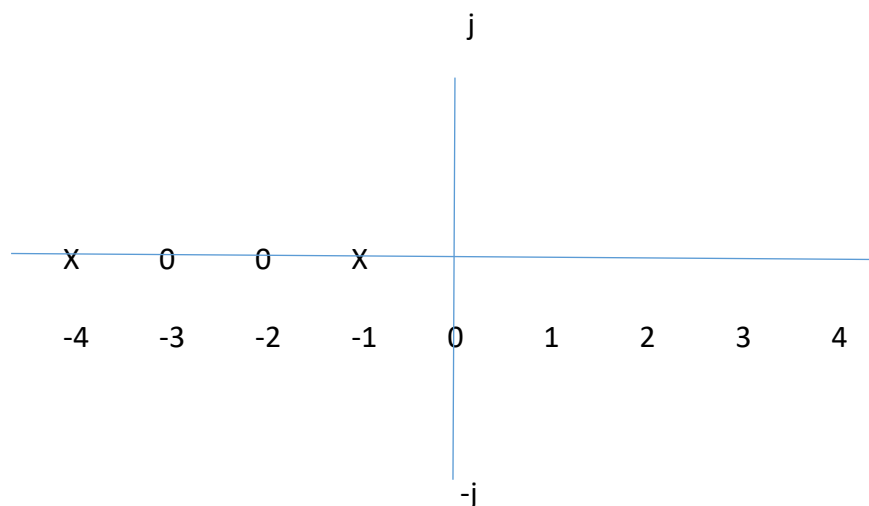
$$G(s) = \frac{(S + 2)(S + 3)}{(S + 1)(S + 4)} \quad (S + 2)(S + 3) = 0 \quad \mathbf{S = -2 \quad \& \quad S = -3}$$

$$G(s) = \frac{(S + 2)(S + 3)}{(S + 1)(S + 4)} \text{ Now find the Poles}$$

$$G(s) = \frac{(S + 2)(S + 3)}{(S + 1)(S + 4)} \quad (S + 1)(S + 4) = 0 \quad \mathbf{S = -1 \quad \& \quad S = -4}$$

Zero = -2, -3 Poles = -1, -4

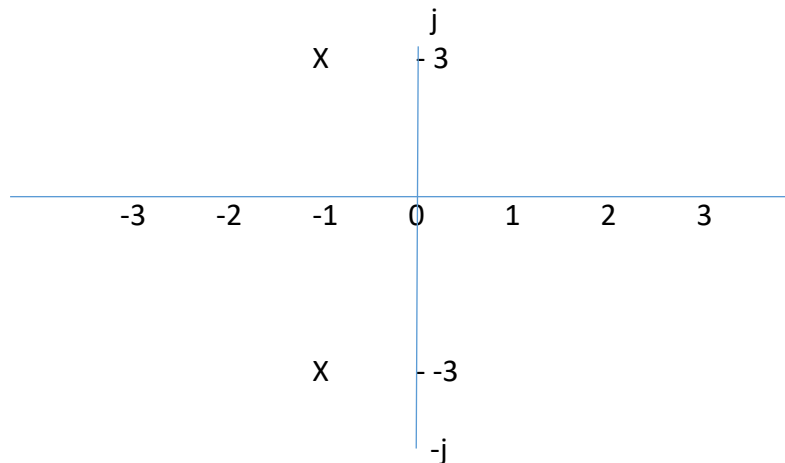
Pole Zero Diagram



When all the POLES (X's) are in the left hand plane, (left hand side of the Diagram) we can deter that the system is Stable. (Cook, 2020-2021)

Example 4:

$$(-1+3j) (-1-3j)$$



Complex Poles are always in conjugate pairs.

The response from a system with a conjugate pair in the LEFT hand plane would be a damped system: an Oscillating system that dies away to a steady state.

The response from a system with a conjugate pair in the RIGHT hand plane would be an Oscillating system that gradually gets larger and does not reach a steady state.

2.3 Routh-Hurwitz

Another method of establishing stability within a closed loop transfer function is the Routh-Hurwitz Criterion.

Calculating the roots of a system for anything larger than a second order polynomial is very time consuming. So in order to determine the stability of a system without solving for the roots directly we use the Routh-Hurwitz Criterion and the Routh Array.

The Routh Array is a way of determining if all the roots lie in the Left Hand Plane of the S Plane and assess stability by looking at the coefficients of the polynomial.

When you are trying to assess the stability of a large order polynomial the first thing you should notice about the coefficients is their sign.

$$G(s) = \frac{1}{s^4 + 3s^3 - 5s^2 + s + 2}$$

If all of the signs are not the same, you can say immediately that the system is unstable. In this case the coefficients are both Positive and Negative so the system is unstable.

If you can't see if the system is stable, we can use the Routh Hurwitz Criterion and fill in a Routh Array. The Routh Array is a table that can be filled in with coefficients of your polynomial and using a few simple rules. The number of rows depend on the order of the polynomial.

The easiest way of explaining this theory is by using an example:

Example I:

$$\text{Transfer Function } T(s) = \frac{5S^2 + 2S + 1}{3S^3 + 4S^2 + 2S + 3}$$

(We are only interested in the denominator, as this is the characteristic equation for this Transfer Function)

Solution:

(Starting with the highest S^3
value for S in the equation S^2
write those down the left S^1
side of the Array S^0
in descending order)

(Then in the top row we S^3 3 2
start inputting our S^2
coefficients, starting with S^1
the first and then every S^0
other coefficient)

(Then in the second row
we input our remaining
coefficients, starting with
the second and then the
remaining coefficients)

$$\begin{array}{rcl} S^3 & 3 & 2 \\ S^2 & 4 & 3 \\ S^1 & & \\ S^0 & & \end{array}$$

(We now need to find the
values of $X^1 X^2$ and Y^1, Y^2)

$$\begin{array}{rcl} S^3 & 3 & 2 \\ S^2 & 4 & 3 \\ S^1 & X^1 & X^2 \\ S^0 & Y^1 & Y^2 \end{array}$$

Starting with the values for the X's

$$\begin{array}{rcl} S^3 & 3 & 2 \\ S^2 & 4 & 3 \\ S^1 & X^1 & X^2 \\ S^0 & & \end{array}$$

$$X^1 = \frac{(4 * 2) - (3 * 3)}{4} = \frac{8 - 9}{4} = \frac{-1}{4} = -0.25$$

S ³	3	2	0	(When there is nothing there we use the value of zero)
S ²	4	3	0	
S ¹	X¹	X²		
S ⁰				

$$X^2 = \frac{(4 * 0) - (3 * 0)}{4} = \frac{0 - 0}{4} = \frac{0}{4} = 0$$

(We can now put in our
values for X¹, X² and X³)

S ³	3	2	0
S ²	4	3	0
S ¹	-0.25	0	0
S ⁰			

Now the values for the Y's

S ³	3	2	0
S ²	4	3	0
S ¹	-0.25	0	0
S ⁰	Y¹	Y²	Y³

$$Y^1 = \frac{(-0.25 * 3) - (4 * 0)}{-0.25} = \frac{-0.75 - 0}{-0.25} = \frac{-0.75}{-0.25} = 3$$

S^3	3	2	0
S^2	4	3	0
S^1	-0.25	0	0
S^0	γ^1	γ^2	γ^3

$$\gamma^2 = \frac{(-0.25 * 0) - (4 * 0)}{-0.25} = \frac{0 - 0}{-0.25} = \frac{0}{-0.25} = 0$$

(We can now put in our

values for γ^1 , γ^2 and γ^3)

S^3	3	2	0
S^2	4	3	0
S^1	-0.25	0	0
S^0	3	0	0



(Like previously, we use a zero

here instead of leaving blank)

We can now determine the stability of this system from the calculated values that have been put into the Routh Array below:

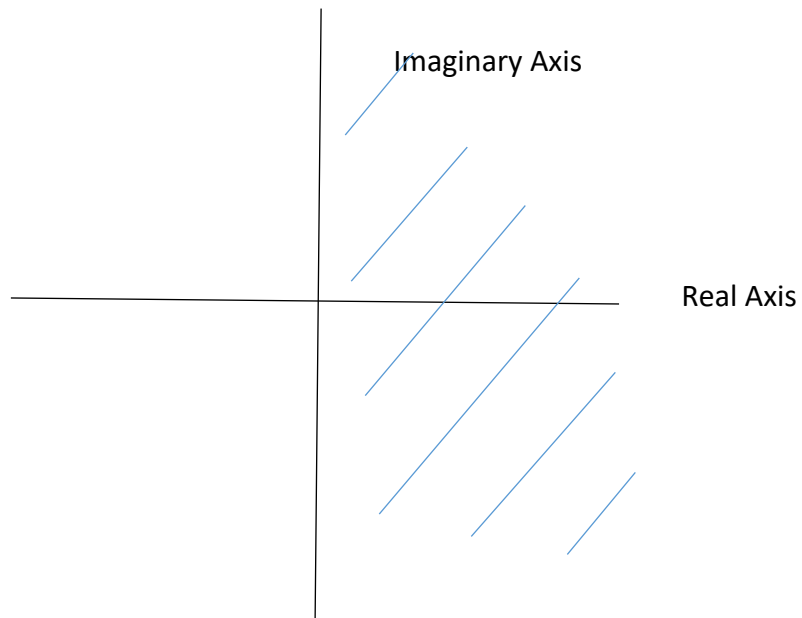
s^3		3	2	0
s^2		4	3	0
s^1		-0.25	0	0
s^0		3	0	0

(We are only interested in the first column at this point)

To determine the stability we need to study the number of sign changes in this first column of the array.

	s^3		3	(positive)
(Sign change	s^2		4	(positive)
here and	s^1		-0.25	(negative)
here)	s^0		3	(positive)

Two sign changes in the first column of the array indicates two roots lying in the Right Half of the S-Plane. Positive real part.



(Chinmaya, 2014)

Example 2:

$$\text{Given the characteristic equation} = 10S^4 + 9S^3 + 3S^2 + 6S + 9 = 0$$

Solution:

(Starting with the highest S^4
value for S in the equation S^3
write those down the left S^2
side of the Array S^1
in descending order) S^0

(Then in the top row we S^4 10 3 9
start inputting our S^3
coefficients, starting with S^2
the first and then every S^1
other coefficient) S^0

(Then in the second row S^4 10 3 9
we input our remaining S^3 9 6 0  (if there is no other coefficient we
coefficients, starting with S^2 put in a zero)
the second and then the S^1
remaining coefficients) S^0

(We now need to find the
values of $X^1X^2X^3$,
 $Y^1Y^2Y^3$ and $Z^1Z^2Z^3$)

S^4	10	3	9
S^3	9	6	0
S^2	X^1	X^2	X^3
S^1	Y^1	Y^2	Y^3
S^0	Z^1	Z^2	Z^3

Starting with the values for the X's

S^4	10	3	9
S^3	9	6	0
S^2	X^1	X^2	X^3
S^1			
S^0			

$$X^1 = \frac{(9 * 3) - (10 * 6)}{9} = \frac{27 - 60}{9} = \frac{-33}{9} = -3.67$$

S^4	10	3	9
S^3	9	6	0
S^2	X^1	X^2	X^3
S^1			
S^0			

$$X^2 = \frac{(9 * 9) - (10 * 0)}{9} = \frac{81 - 0}{9} = \frac{81}{9} = 9$$

(We can now put in our	S ⁴	10	3	9	(We use zero here, if it was a
values for X ¹ , X ² and X ³)	S ³	9	6	0	larger order polynomial there
	S ²	-3.67	9	0	← may be another column and
	S ¹				further calculations in same way)
	S ⁰				

Now the values for the Y's

S ⁴	10	3	9
S ³	9	6	0
S ²	-3.67	9	0
S ¹	Y ¹	Y ²	Y ³
S ⁰			

$$Y^1 = \frac{(-3.67 * 6) - (9 * 9)}{-3.67} = \frac{-22.02 - 81}{-3.67} = \frac{-103.02}{-3.67} = 28.07$$

S ⁴	10	3	9
S ³	9	6	0
S ²	-3.67	9	0
S ¹	Y ¹	Y ²	Y ³
S ⁰			

$$Y^2 = \frac{(-3.67 * 0) - (9 * 0)}{-3.67} = \frac{0 - 0}{-3.67} = \frac{0}{-3.67} = 0$$

(We can now put in our
values for Y^1 , Y^2 and Y^3)

S^4	10	3	9	
S^3	9	6	0	
S^2	-3.67	9	0	(Like previously, we use a zero
S^1	28.07	0	0	← here instead of leaving blank)
S^0				

Now the values for the Z's

S^4	10	3	9
S^3	9	6	0
S^2	-3.67	9	0
S^1	28.07	0	0
S^0	Z^1	Z^2	Z^3

$$Z^1 = \frac{(28.07 * 9) - (-3.67 * 0)}{28.07} = \frac{252.63 - 0}{28.07} = \frac{252.63}{28.07} = 9$$

S^4	10	3	9
S^3	9	6	0
S^2	-3.67	9	0
S^1	28.07	0	0
S^0	Z^1	Z^2	Z^3

$$Z^2 = \frac{(28.07 * 0) - (-3.67 * 0)}{28.07} = \frac{0 - 0}{28.07} = \frac{0}{28.07} = 0$$

(We can now put in our values for Z^1 , Z^2 and Z^3)

S^4	10	3	9
S^3	9	6	0
S^2	-3.67	9	0
S^1	28.07	0	0
S^0	9	0	0

We can now determine the stability of this system from the calculated values that have been put into the Routh Array below:

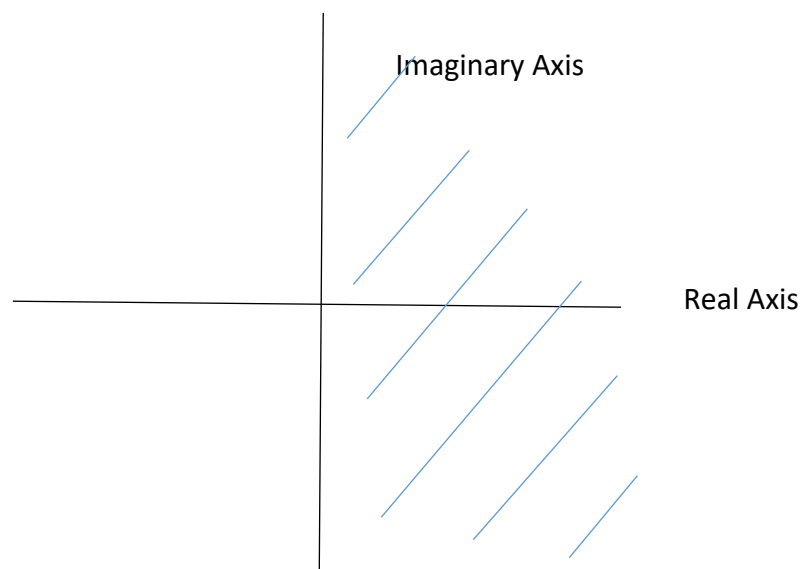
S^4	10	3	9
S^3	9	6	0
S^2	-3.67	9	0
S^1	28.07	0	0
S^0	9	0	0

(We are only interested in the first column at this point)

To determine the stability we need to study the number of sign changes in this first column of the array.

	S^4	10	(positive)
(Sign change	S^3	9	(positive)
here and	S^2	-3.67	(negative)
here)	S^1	28.07	(positive)
	S^0	9	(positive)

Two sign changes in the first column of the array indicates two roots lying in the Right Half of the S-Plane. Positive real part.



Again, similar to the Poles and Zero's technique, if any of the roots are in the right hand plane the system is said to be unstable, if you have any roots on the imaginary axis the system is said to be marginally stable, and if all the roots are in the left hand plane the system is stable. (Ekeeda, 2018)

3 Task 3

Working in partnership with a structural engineer, you are tasked with providing an instrumentation system for monitoring the stress in a load bearing structure. The item in question is made from 355 grade aluminum alloy with T6 Temper. The dimensions of the item are shown below in Figure 1.

It is expected that loads of up to 5kN may be applied in the direction of the long axis as indicated.

It is given that the yield strength of this material is 22MPa, Youngs modulus $E = 10.3\text{GPa}$. The item in question is located outside in an open environment and the distance from the item to the main instrumentation room is 20m.

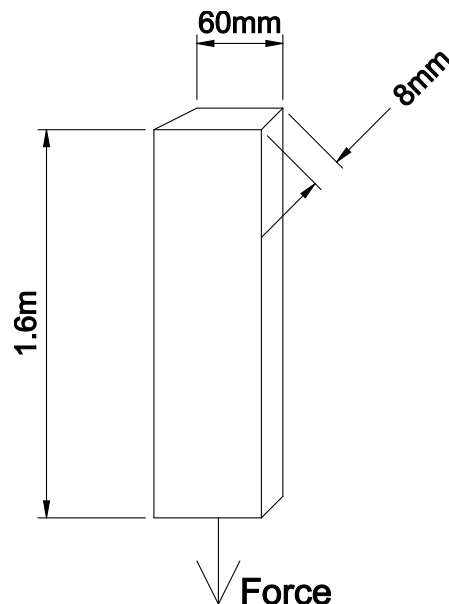
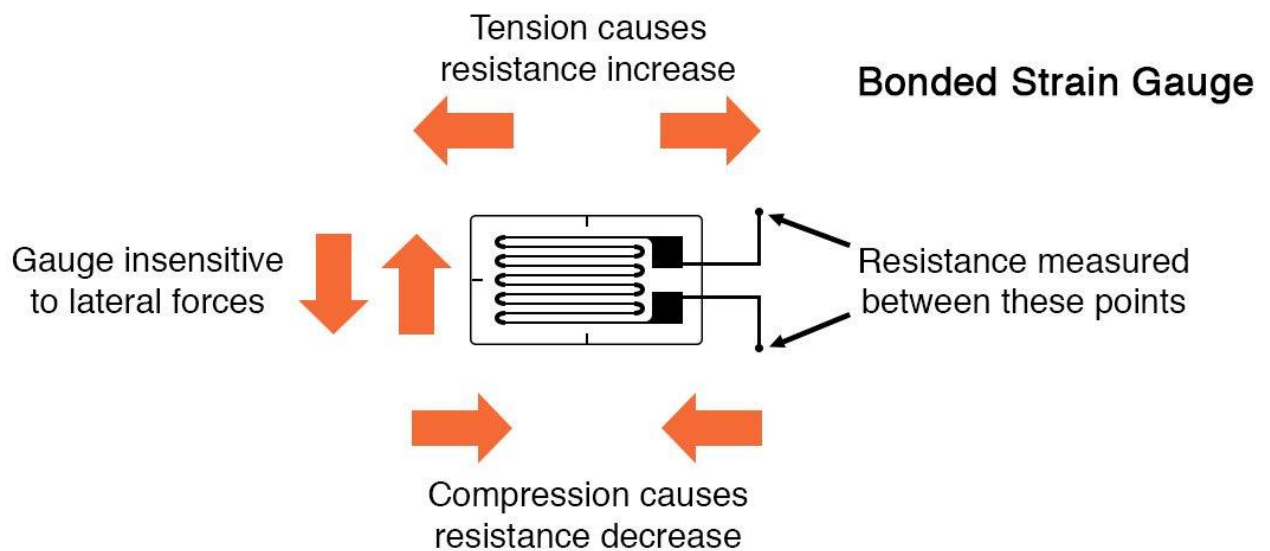


Figure 1

You are asked to provide a short report explaining how you will provide a suitable solution.

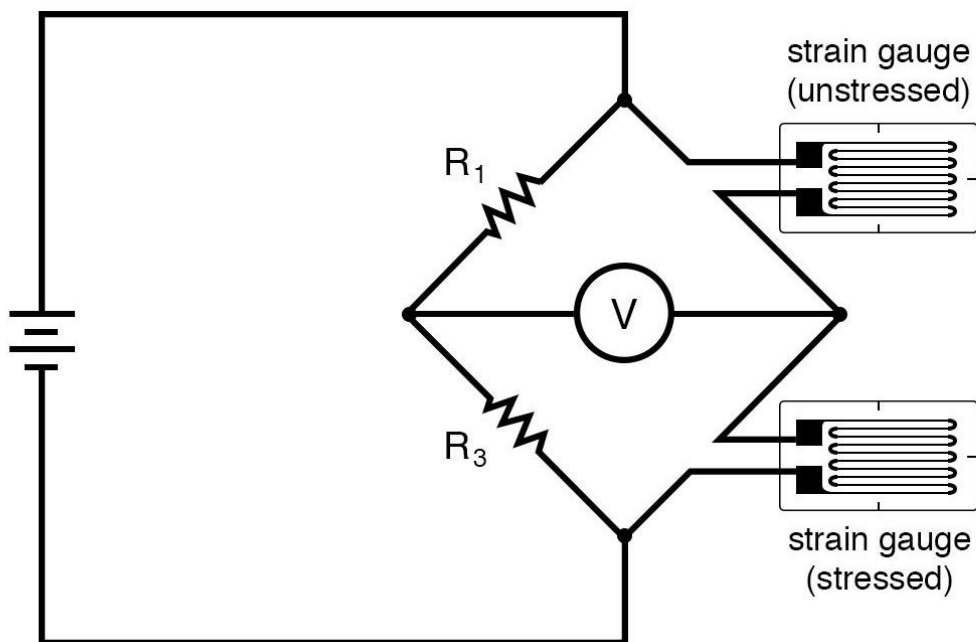
3.1 Operational aspects of the measurement system.

The main component used to measure the stress on the structure will be a strain gauge. Most strain gauges are smaller than a postage stamp, and they look something like this:



Typical strain gauge resistances range from $30\ \Omega$ to $3\ \text{k}\Omega$ (unstressed). This resistance may change only a fraction of a percent for the full force range of the gauge, in order to use the strain gauge as a practical instrument, we must measure extremely small changes in resistance with high accuracy. Such demanding precision calls for a bridge measurement circuit. A strain gauge Wheatstone Bridge Circuit indicates measured strain by the degree of imbalance and uses a precision voltmeter in the centre of the bridge to provide an accurate measurement of that imbalance.

An unfortunate characteristic of strain gauges is that of resistance change with changes in temperature. This is a property common to all conductors, some more than others. Thus, a quarter-bridge circuit works as a thermometer just as well as it does a strain indicator. If all we want to do is measure strain, this is not good. We can transcend this problem, however, by using a "dummy" strain gauge in place of R_2 , so that *both* elements of the rheostat arm will change resistance in the same proportion when temperature changes, thus cancelling the effects of temperature change.

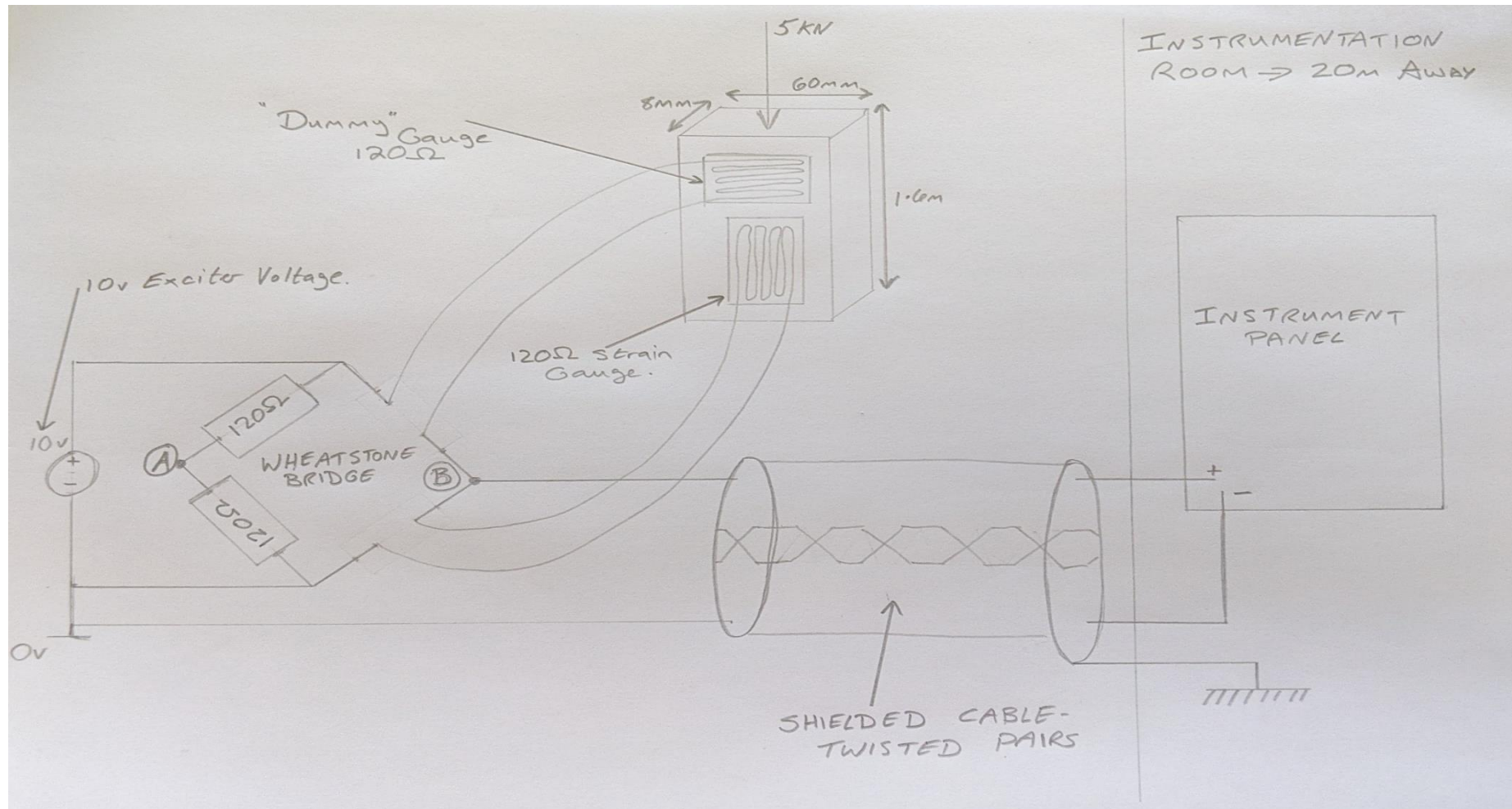


Resistors R_1 and R_3 are of the equal resistance value, and the strain gauges are identical to one another. With no applied force, the bridge should be in a perfectly balanced condition and the voltmeter should register 0 volts. Both gauges are bonded to the same test specimen, but only one is placed in a position and orientation so as to be exposed to physical strain (the *active* gauge). The other gauge is isolated from all mechanical stress and acts merely as a temperature compensation device (the “*dummy*” gauge). If the temperature changes, both gauge resistances will change by the same percentage, and the bridge’s state of balance will remain unaffected. Only a differential resistance (difference of resistance between the two strain gauges) produced by physical force on the test specimen can alter the balance of the bridge.

The fact that the instrumentation room is 20m away from the measurement system is one factor that may be likely to affect the quality of the measurement due to the higher resistance of the cable that this will generate as opposed to if the instrumentation room was closer.

Wire resistance doesn’t impact the accuracy of the circuit as much as before, because the wires connecting both strain gauges to the bridge are approximately equal length. Therefore, the upper and lower sections of the bridge’s rheostat arm contain approximately the same amount of stray resistance, and their effects tend to cancel each other out. However, the length of the cables do pose another problem aside from resistant values, and this issue is Thermal Noise (generated from external machinery) and Shock Noise (generated inside circuitry). To minimise the effect of electrical Noise we use Twisted Pairs of conductors within our circuit, along with a copper shield to the cables that is connected to the GND at the Instrumentation Panel. (Author, Strain Gauges, 2021)

3.2 A Sketch of the proposed measurement system:



3.3 Measurement Circuit Calculations:

3.3.1 Stress (σ)

$$Area = Length * Width$$

$$60mm * 8mm = Area \text{ mm}^2$$

$$60mm * 8mm = 480mm^2$$

$$SI \text{ Unit Area} = \text{meters}^2$$

$$\therefore Area = 0.00048m^2$$

$$Stress (\sigma) = \frac{Load}{Area}$$

$$Stress (\sigma) = \frac{5KN}{0.00048m^2}$$

$$\therefore Stress (\sigma) = 10,416,666.67KPa$$

$$\text{or } Stress (\sigma) = 10.42MPa$$

3.3.2 Strain (ϵ)

$$\text{Strain } (\epsilon) = \frac{\text{Stress } (\sigma)}{\text{Youngs Modulus } (E)}$$

$$\text{Strain } (\epsilon) = \frac{10.42 \text{ MPa}}{10.3 \text{ GPa}}$$

$$\text{Strain } (\epsilon) = \frac{10.42 * 10^6}{10.30 * 10^9}$$

$$\text{Strain } (\epsilon) = 1.01165[10]^{-3}$$

3.3.3 Delta Length (ΔL)

$$\text{Strain } (\epsilon) = \frac{\Delta L}{L}$$

$$\text{or Strain} * \text{Length} = \Delta L$$

$$1.01165[10]^{-3} * 1.6 = 1.618640777[10]^{-3}$$

$$\therefore \Delta L = 1.6186 \text{ mm}$$

$$\text{or } \Delta L = 0.0016186 \text{ meters}$$

3.3.4 Delta Resistance (ΔR)

Assumed Values: Gauge Factor is defined by manufacturer, but is typically 2

Resistors in our Wheatstone bridge are 120Ω

$$\Delta R = \text{Gauge Factor} * \varepsilon * R$$

$$2 * 1.01165[10]^{-3} * 120 = 0.242796\Omega$$

$$\therefore \Delta R = 0.242796\Omega$$

3.3.5 Output Voltage

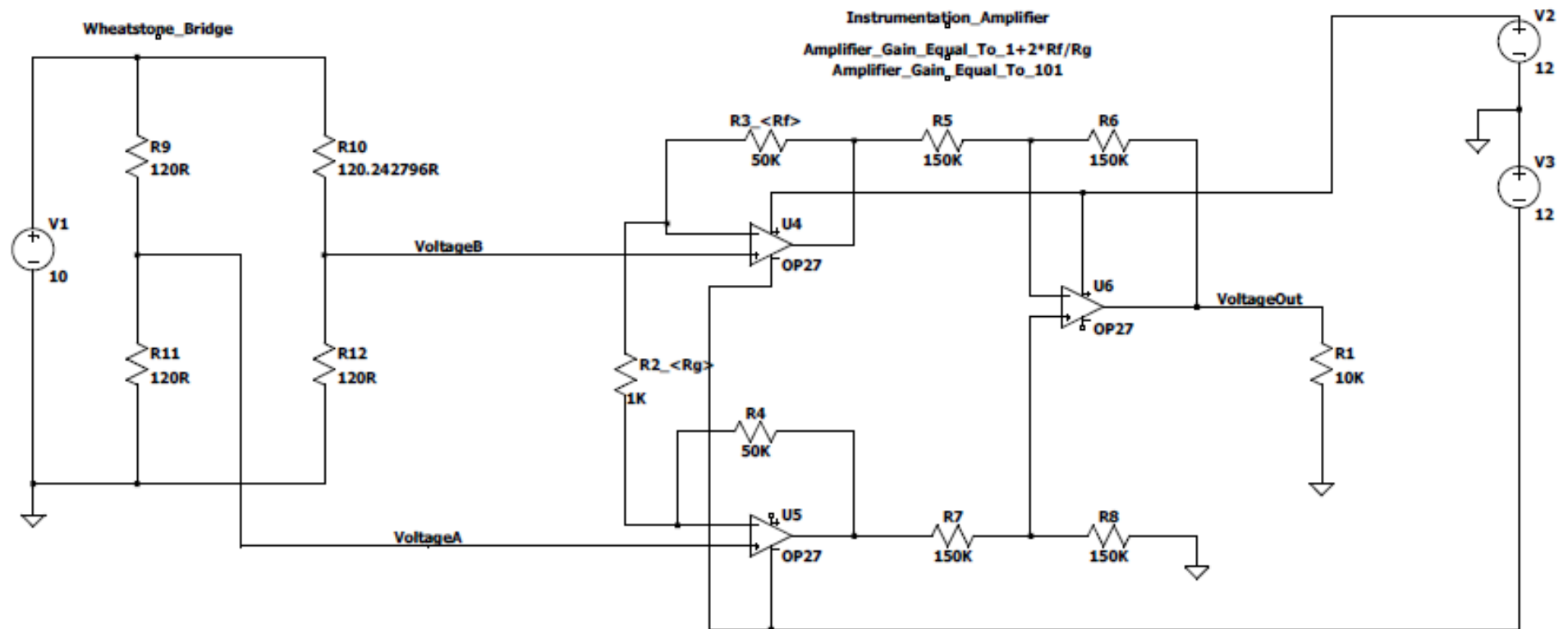
$$\text{Exciter Voltage} = 10 \text{ Volts}$$

$$\text{Voltage at A} = 5 \text{ Volts}$$

$$\text{Voltage at B} = \frac{120 * 10}{120 + 120.242796} = 4.994946862 \text{ Volts}$$

$$\Delta V = 0.005053138 \text{ Volts}$$

3.4 Circuit Diagram of the proposed measurement system:



4 Task 4

A system of two masses M_1 and M_2 are located on a frictionless surface as shown in figure 2 below.

They are connected with two springs (k_1 and k_2) and two viscous dampers (c_1 and c_2) as shown.

- Determine an expression for the transfer function for the system - take $f(t)$ as the input and the displacement of M_2 $x_2(t)$ as the output.
- Using the following values produce a Simulink model for the system: demonstrate and comment on the system response when a unit step is applied as the input force.

$$M_1 = 1.5 \text{ kg}, M_2 = 1 \text{ kg}, k_1 = 1, k_2 = 2, c_1 = 0.3, c_2 = 0.5.$$

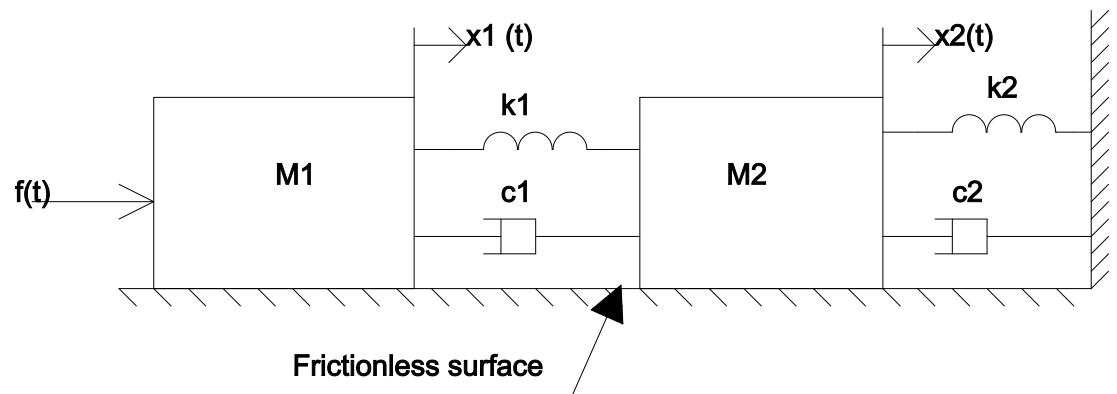


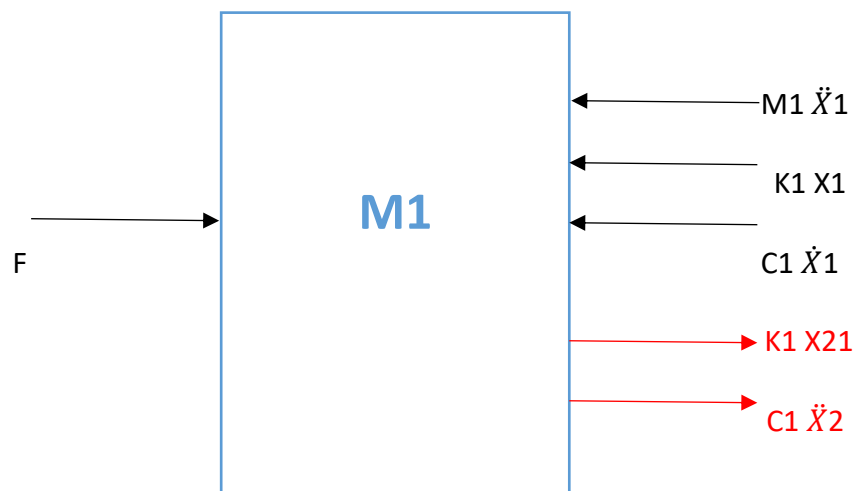
Figure 2

4.1 Expression for a Transfer Function

4.1.1 Free Body Diagram Black: With M2 held, Red: M1 held and M2 moved to the right.

$$\dot{X} = \frac{dx}{dt}$$

$\ddot{X}$ = Double Differentiation



(Rule of thumb: Everything to the right is positive and everything to the left is negative)

$$F - M1\ddot{X}_1 - K1X1 - C1\dot{X}_1 + K1X2 + C1\dot{X}_2 = 0$$

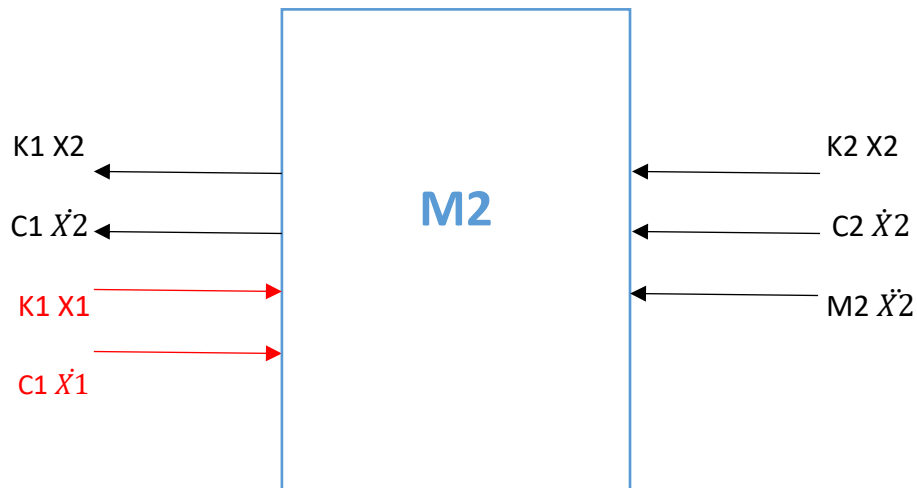
$$M1\ddot{X}_1 = F - K1X1 - C1\dot{X}_1 + K1X2 + C1\dot{X}_2$$

$$\ddot{X}_1 = \frac{1}{M1} (F - K1X1 - C1\dot{X}_1 + K1X2 + C1\dot{X}_2) = 1st Value$$

Shorthand Expression for use with MATLAB:

$$x1dd=(1/m1)(f-k1x1-c1x1d+k1x2+c1x2d)$$

4.1.2 Free Body Diagram Black: M1 held and M2 moved to the right, Red: M2 held and M1 moved to the right.



$$K_1 X_1 + C_1 \dot{X}_1 - K_1 X_2 - C_1 \dot{X}_2 - K_2 X_2 - C_2 \dot{X}_2 - M_2 \ddot{X}_2 = 0$$

$$M_2 \ddot{X}_2 = K_1 X_1 + C_1 \dot{X}_1 - K_1 X_2 - C_1 \dot{X}_2 - K_2 X_2 - C_2 \dot{X}_2$$

$$\ddot{X}_2 = \frac{1}{M_2} (K_1 X_1 + C_1 \dot{X}_1 - K_1 X_2 - C_1 \dot{X}_2 - K_2 X_2 - C_2 \dot{X}_2) = 2^{nd} \text{ Value}$$

Shorthand Expression for use with MATLAB:

$$x2dd = (1/m2)(k1x1 + c1x1d - k1x2 - c1x2d - k2x2 - c2x2d)$$

4.2 MATLAB Model

4.2.1 MATLAB Script.

The screenshot displays the MATLAB Online R2021a web interface. The browser address bar shows `matlab.mathworks.com`. The top navigation bar includes tabs for HOME, PLOTS, APPS, EDITOR, PUBLISH, FILE VERSIONS, and VIEW. The EDITOR tab is active, showing a script named `AssignmentConstantValues.m` with the following code:

```
1 % Constants for the Mass Spring Damper with two masses
2 m1 = 1.5;
3 m2 = 1;
4 k1 = 1;
5 k2 = 2;
6 c1 = 0.3;
7 c2 = 0.5;
```

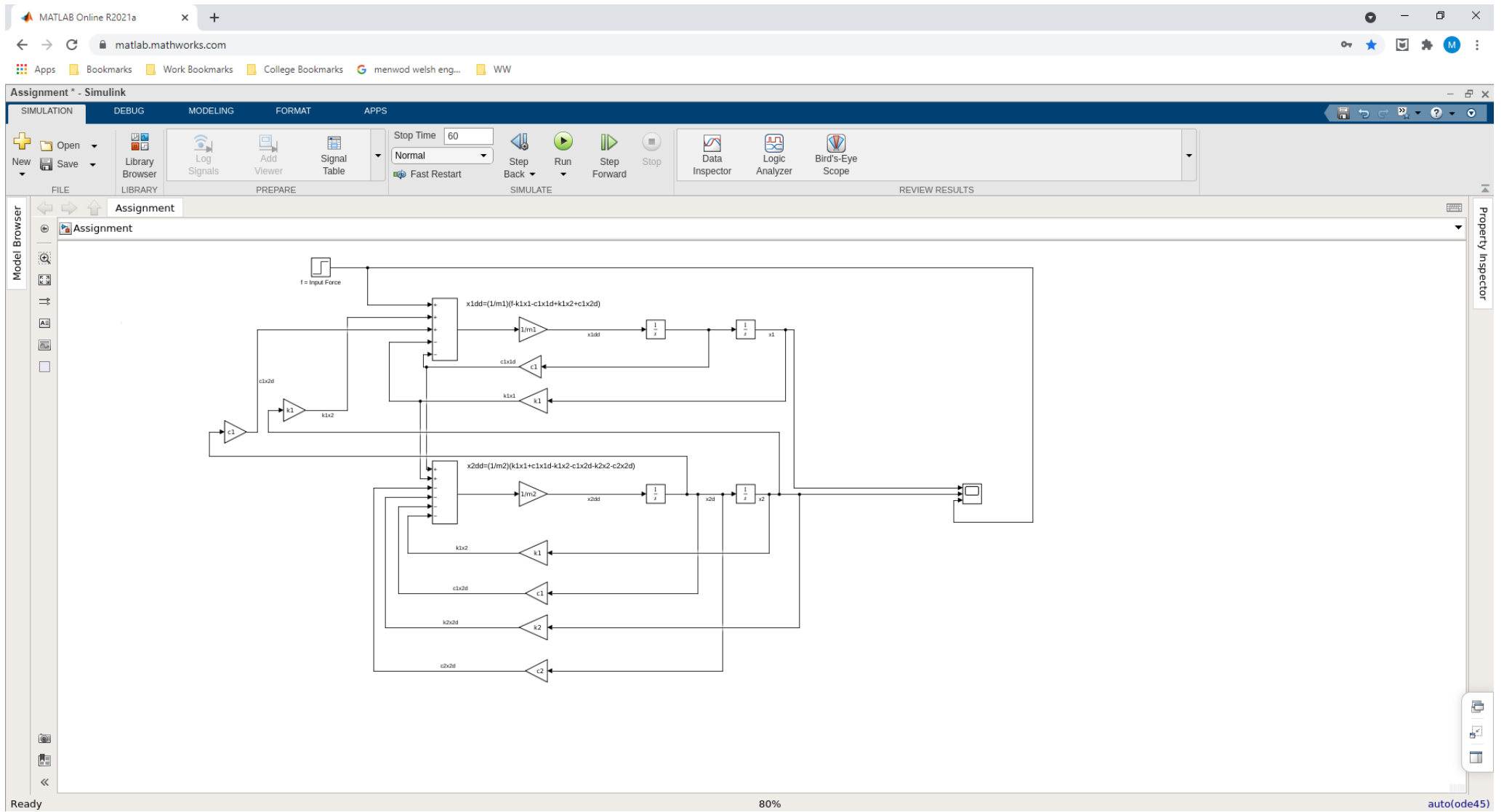
The left sidebar contains a 'Current Folder' pane showing a file explorer view of the `MATLAB Drive` directory. The 'Workspace' pane below it displays a table of variables:

Name	Value	Size	Class
c1	0.3000	1×1	double
c2	0.5000	1×1	double
k1	1	1×1	double
k2	2	1×1	double
m1	1.5000	1×1	double
m2	1	1×1	double
out	1×1 Simula...	1×1	Simulink:Sl...

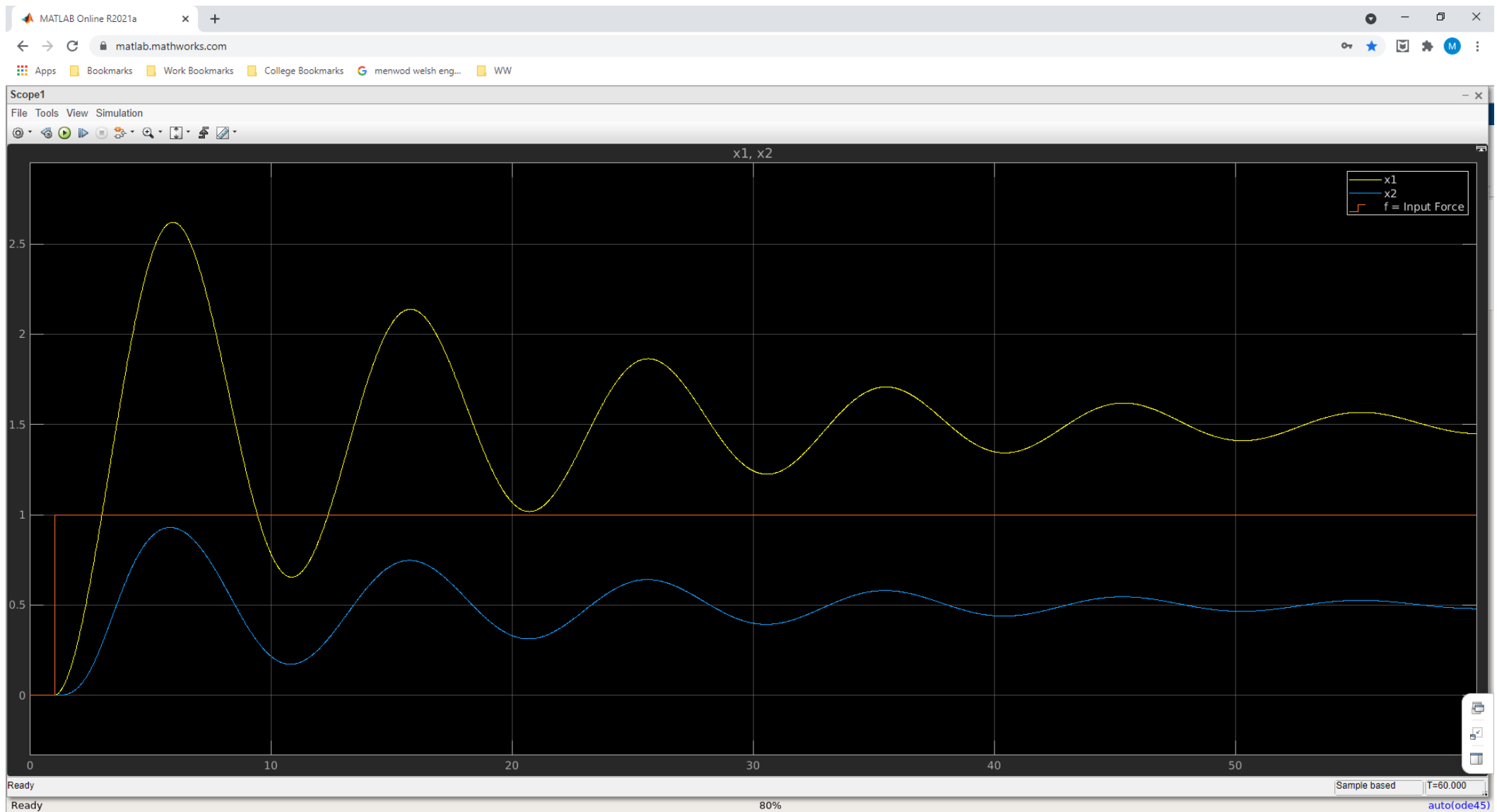
The bottom pane is the 'Command Window', which shows the command `>> AssignmentConstantValues` has been executed successfully.

The status bar at the bottom right indicates the file encoding is UTF-8, line endings are CRLF, the file type is script, and the cursor is at line 7, column 10.

4.2.2 MATLAB Simulink Model.



4.2.3 MATLAB Output Scope of the system.



4.2.4 Analysing the data from the system.

From the above we can see that a unit step input is applied as the input force (Red Line). This is, that the input changes from 0 to 1 in a very short time and the first mass is essentially given a “push” when the step input is applied. As the two masses are connected with springs and dampers the second mass then bounces as the inertia from the first mass causes it to also oscillate back and forth. The unit step response of the system (Yellow Line X1, Blue Line X2), shows an oscillating output as the masses move back and forth.

The scope is demonstrating the change in displacement of the two masses and how they move when the step input is applied.

5 References

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